Question 6

According to quantum mechanics the dynamics of electrons can either be treated as the motion of particles or the propagation of waves.

a) Give two (or more) examples (relevant to the lectures) which show the particle character.
b) Give two (or more) examples (relevant to the lectures) which show the wave character or electrons

Question 7

Consider electrons with an effective mass m* and energy E.

a) The electrons are confined in a layer with thickness D. In the x and y directions the electrons can move freely. What should the thickness D be so that the electron system is two-dimensional?

b) Now the electrons are confined in a cube with thickness, length and width equal to D. What is the value of D so that the energy of the lowest 0-dimensional state is equal to E?

c) Can you give a few examples of 2-dimensional electron systems? And 1-dimensional and 0-dimensional systems?

Question 8

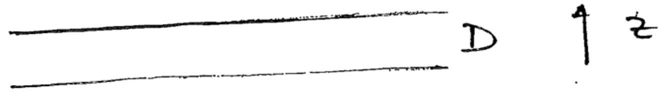
4 a) Describe the operation of a single electron transistor. Give a schematic diagram of a single electron transistor. Show the distribution of charges when the current through the transistor is off, and when it is on.

b) Discuss what is needed for a good operation of the transistor. Consider role of capacitance, temperature, resistance and possible other effects which can influence the operation.

c) Do you think that single electron transistors can replace conventional field effect transistors? Give some reasons why, or why not.

question 7

3) (4)

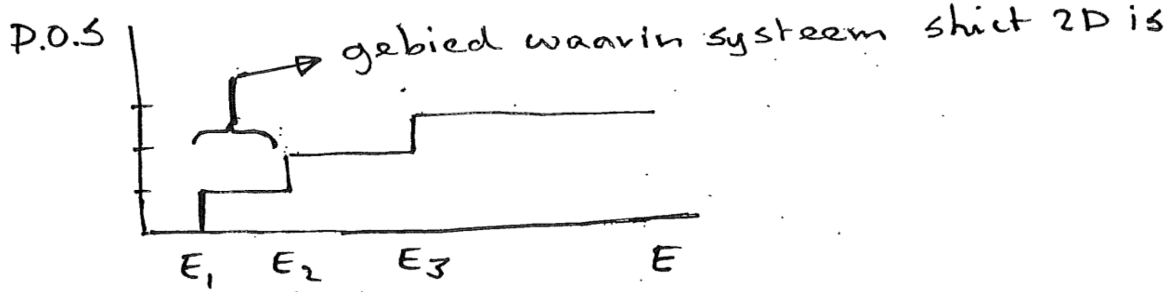


concentratie N_D

$$E = E_{kin} = \frac{\hbar^2}{2m^*}\left(k_x^2 + k_y^2 + k_z^2\right)$$

Door opsluiting in z richting met "harde wand" potentiaal $\rightarrow$ golffunctie $= 0$ bij de wanden $\rightarrow$ $k_z = \frac{\pi N}{D}$ met N: integer.

Density of states:



$$E_1 = \frac{\hbar^2}{2m^*} \cdot \frac{\pi^2}{D^2} \qquad E_2 = \frac{\hbar^2}{2m^*} \cdot \frac{4\pi^2}{D^2} \qquad E_3 = \frac{\hbar^2}{2m^*} \frac{9\pi^2}{D^2}$$

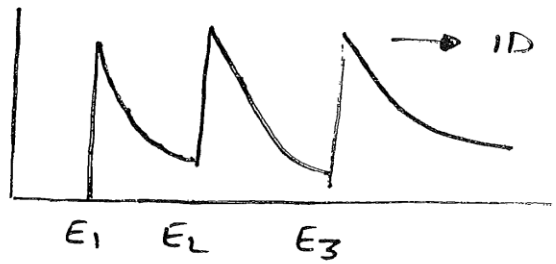
2D DOS : $\frac{m^*}{\pi\hbar^2}$ (inclusief spin ontaarding)

Voor strict 2D moet dus gelden

$$\frac{m}{\pi\hbar^2}(E_2 - E_1) > \frac{N_D}{D}$$

Hieruit volgt een voorwaarde voor D

c) idem maar nu voor 1D (5)



→ 1D toestandsdichtheid
zie : Spears.

Fermi energie moet in dit gebied liggen

a) quantum tunnelen door gate oxide
quantum tunnelen door kanaal tussen source en drain

b) de standaard FETs maken geen gebruik van de Coulomb blockade, dus het feit dat de electrostatische energie op een discrete manier verandert als het aantal electronen N verandert.

c) zie : lecture notes

d) voor : klein, werkt al bij weinig electronen
gevoelig

tegen : * bij kamertemperatuur is er nog steeds genoeg thermische energie om "random" electronen te laten tunnelen
* de electrische "ladingsomgeving" moet gecontroleerd worden ~~tot op~~ beter dan een electron lading
* de RC tijd is groot ($R > h/e^2$)

Question 4:

A semiconductor wire has a circular cross section (in y and z directions), and is extended in the x direction. The electrostatic potential of the electrons is written as $V(r) = -a r^2$ with $r^2 = y^2 + z^2$ (Note that this potential can be written as the sum of potentials which only depend on y or z).

a) Write down the (time independent) Schrodinger equation which describes the electronic states in the wire (Assume that the electrons have an effective mass m*).

Due to the confinement in the radial (y,z) direction discrete electronic states are formed.

b) Make a drawing of the energy levels (indicate what the level spacing is and if there are degeneracies)

Due to the extended electronic states in the x-direction, the density of states of the wire consists of a series of 1-dimensional subbands.

c) Make a schematic drawing of the density of states as a function of energy.

The electronic states are occupiaed up to the Fermi energy Ef.

d) For which values of the Fermi energy are the electronic states in the wire strictly 1 dimensional?

question 4

(E)

a)

$$H\psi = E\psi$$

$$H = -\frac{h^2}{2m^*}\nabla^2\psi - eV(r)$$

$$= -\frac{h^2}{2m^*}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + ea(\cancel{x}^2 + y^2)$$

$$= \underbrace{-\frac{h^2}{2m^*}\frac{\partial^2\psi}{\partial x^2}}_{H_x} \underbrace{-\frac{h^2}{2m^*}\frac{\partial^2\psi}{\partial y^2} + eay^2}_{H_y} \underbrace{-\frac{h^2}{2m^*}\frac{\partial^2\psi}{\partial z^2} + eaz^2}_{H_z}$$

assume $\psi = \psi(x)\,\psi(y)\,\psi(z)$

in ~~x~~ an z direction : harmonic oscillator

inx direction : free motion

(b) (F)

harmonic oscillator:

$$H = \frac{1}{2} m^* v_y^2 + e a y^2$$

$$\omega = \sqrt{\frac{2ea}{m^*}}$$

energy in y direction:

$$E_y = (n_y + \tfrac{1}{2})\hbar\omega \qquad n_y = 0, 1, 2, \ldots$$

energy in z direction:

$$E_z = (n_z + \tfrac{1}{2})\hbar\omega \qquad n_z = 0, 1, 2, \ldots$$

total energy in y an z direction

$3\hbar\omega$ — $n_z=1, n_y=1$ — $n_z=0\ n_y=2$ — $n_z=2\ n_y=0$

$2\hbar\omega$ — $n_z=0, n_y=1$ — $n_z=1, n_y=0$

$\hbar\omega$ — $n_z, n_y=0$

0

G

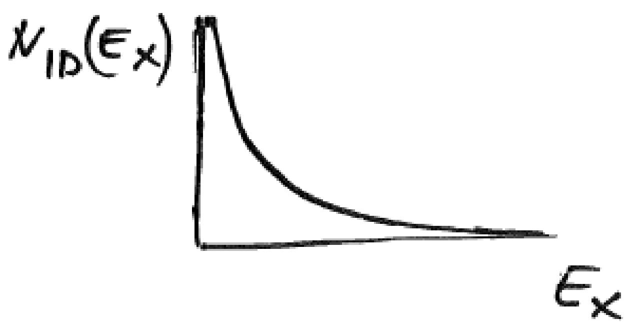
in x direction:

$$\psi(x) = \exp(ikx)$$

$$E_x = -\frac{\hbar^2}{2m^*}\frac{\partial^2\psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m^*}$$

1 dimensional DOS:

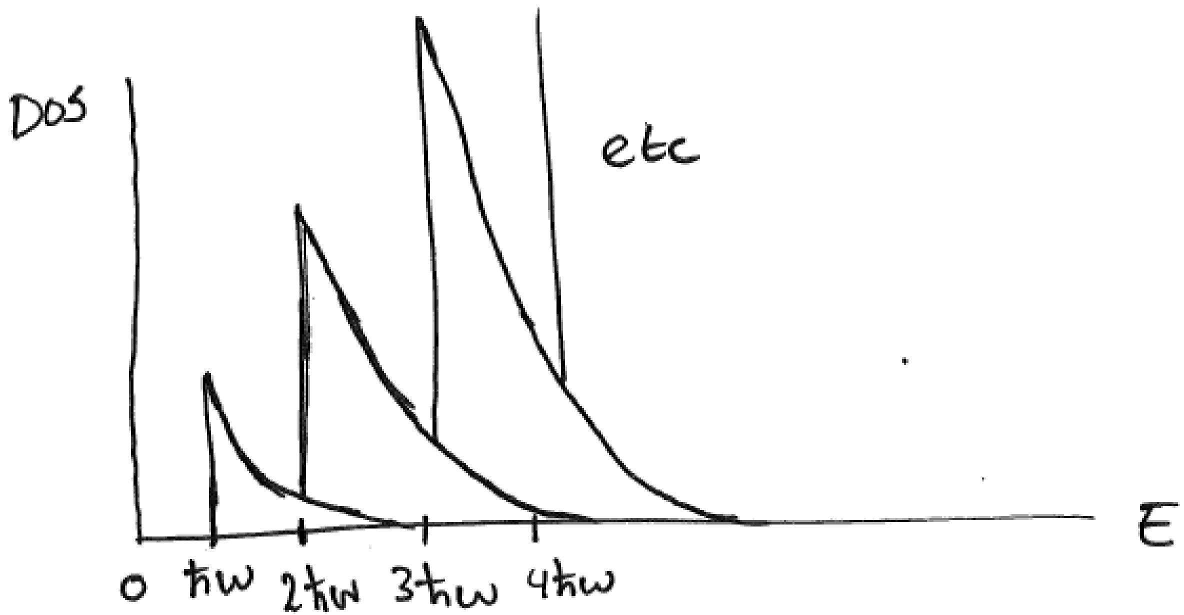
$$N_{1D}(E_x) = \frac{\sqrt{2}\sqrt{m^*}}{\pi\hbar}\frac{1}{\sqrt{E_x}}$$



(H)

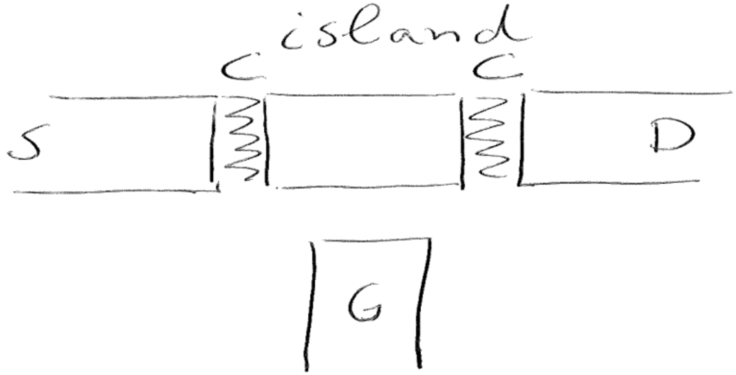
total DOS :

$$E = E_x + E_y + E_z$$



strictly 1 dimensional when

$$\hbar\omega < E_F < 2\hbar\omega$$



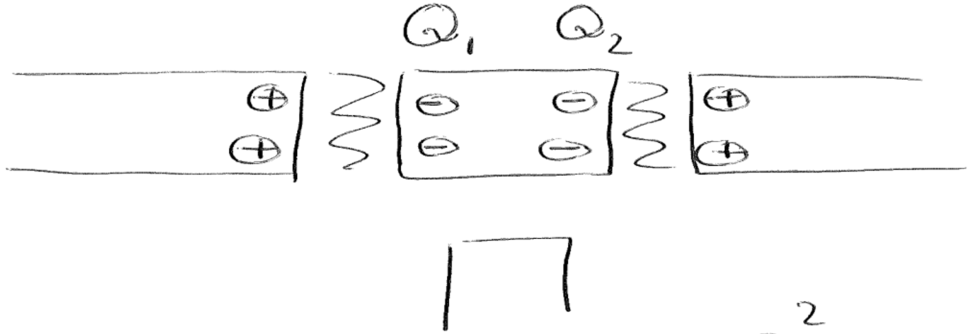
SET

<u>situation 0</u>

neutral island

convention : ⊖ or ⊕ : 1/4 electroncharge

<u>situation 1</u> : neutral island after tunneling of 1 electron



Charging energy : $\frac{1}{2}\frac{Q_1^2}{C} + \frac{1}{2}\frac{Q_2^2}{C}$

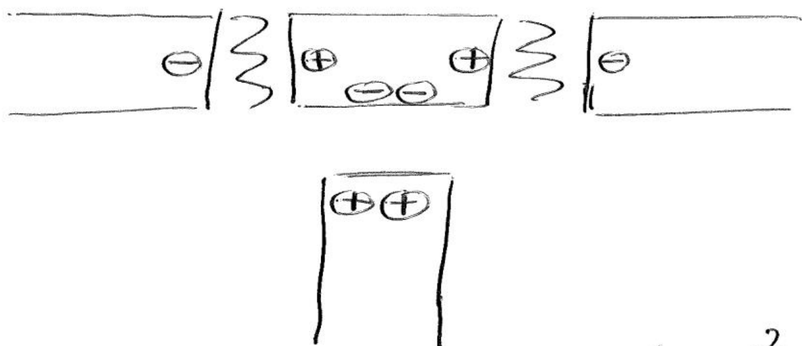
$$= \frac{1}{8}\frac{e^2}{C} + \frac{1}{8}\frac{e^2}{C} = \frac{1}{4}\frac{e^2}{C}$$

energy change :

$$\Delta E = \frac{1}{4}\frac{e^2}{C} - 0 = \frac{1}{4}\frac{e^2}{C} \longrightarrow \text{Coulomb blockade}$$

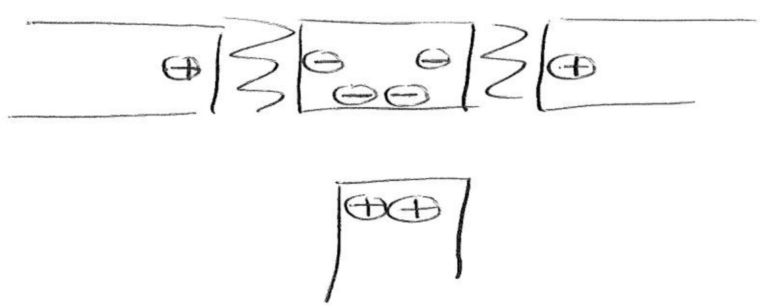
situation 2 gate bias $= +\frac{1}{2}e$

a) before tunneling:

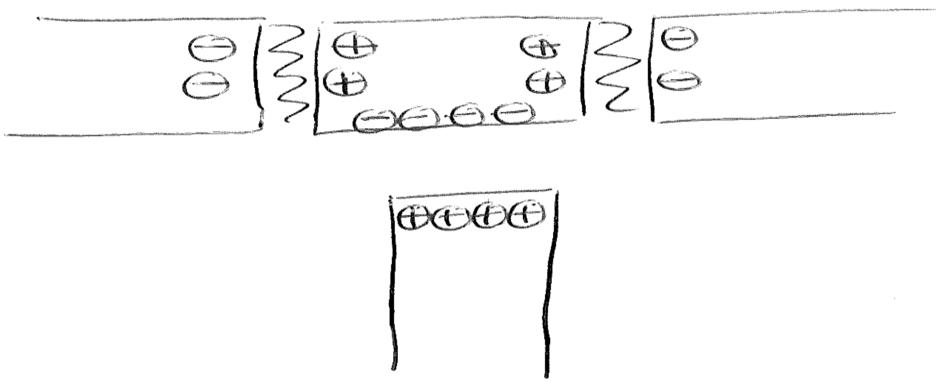


charging energy: $\frac{1}{2}\frac{\left(\frac{1}{4}e\right)^2}{C} + \frac{1}{2}\frac{\left(\frac{1}{4}e\right)^2}{C} = \frac{1}{16}\frac{e^2}{C}$

b) after tunneling



charging energy the same $\longrightarrow$ No Coulomb blockade situation 3 gate bias = + 1 e



the system minimizes its charging energy by the tunneling of an electron → back to situation 0